Visualization Techniques For Four-Stokes Parameter Polarization

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ABSTRACT

The human visual system can barely detect polarized light. This paper outlines a methodology for displaying and visualizing polarization profiles of natural, daylight scenes. A pseudo color scheme, based on the Poincare sphere, is developed for encoding the various polarization parameters in a scene. A mathematical model of the polarization state of skylight is given. A pictorial representation of the polarization state of the sky, using the Poincare colorization methodology, is presented. Using the colorization methodology, a visual correlation is given between the polarization states of light reflected from a target and the solar illumination producing it.

INTRODUCTION

The human visual system can barely detect polarized light. Some observers can determine, through sighting Haidinger's brush[1], that skylight is polarized. However, the human visual system is incapable of determining the complete polarization state of a light beam. Of course, we have instruments and methods that allow us to determine all the polarization parameters of a polarized beam of light. But how does one pictorially display all these parameters, especially if they vary spatially, as in a daylight scene? This paper is dedicated to our current work, which attempts to simply and conveniently solve this problem. The proposed solution involves a pseudo-color scheme that closely relates to the Poincaré sphere representation of polarization, developed by Henri Poincaré in 1892[2]. The surface of this sphere is commonly used to give a pictorial representation of all the possible polarization states of completely polarized light. Each point on the surface of the sphere corresponds to a unique polarization state. Right circular polarization is represented at the north pole and left circular polarization is represented at the south pole of the sphere. Linear polarization of all possible orientations is represented along the equator of the sphere. Elliptical polarization of all possible azimuth and ellipticity angles and handedness are represented at other points on the surface of the sphere.

Typically, only the surface of the Poincaré sphere is used to represent a polarization state[3]. Partially polarized light has been represented by a point moving along the surface of the sphere[4], the point spending more time in the neighborhood of a particular point. Our use of the Poincaré sphere is rather unconventional in that we use the inside of the Poincaré sphere as well as the surface of the sphere. The surface of the sphere corresponds to totally polarized light while the inside of the sphere corresponds to partial polarization, the center of the sphere corresponding to unpolarized light. This approach is in exact agreement with the one-to-one correspondence between the normalized Stokes parameters and the rectangular Cartesian coordinates of a point inside or on the surface of the Poincaré sphere.

Our main purpose in establishing a methodology for displaying and visualizing polarization states is to formulate a complete polarization profile of all objects in a natural, daylight scene. The recording of polarization information from a daylight scene can be accomplished through the use of a digital camera fitted with a filtering system that allows for the calculation of the four Stokes parameters[5].

This paper is a continuation of an extended effort to analyze ground combat vehicle signatures. Previous work[6-7] presented the results of a comprehensive study of the diurnal relationship between the slopes of target facets and edges, time of day and the azimuth angle of specular reflection of sunlight into a horizontal plane; the degree of polarization of reflected daylight from various paints and polarization profiles of diffuse and specular reflections from various paints and objects in daylight using a digital camera. In those previous studies, we showed that a complex index of refraction in the Fresnel reflection coefficients describes the degree of polarization for various paints under unpolarized light. Our goal then, as it is now, is to

understand the passive, visual signatures of targets in terms of phenomenological parameters such as angles of incidence and reflection, polarization angles, material properties, diurnal changes, vehicle geometry and shape, and scene content. Recent technological developments in the digital camera industry supply us with new tools to accomplish our goals and refresh previous interests.

Incident light on ground targets in daylight originates from two primary sources: sunlight and skylight. Sunlight is unpolarized, but diffuse skylight can be highly polarized due to Rayleigh scattering of sunlight[8-9]. Because the polarization state of skylight is different at each point on the celestial sphere, it is very difficult (if not impossible) to replicate this light source in the laboratory. One has no other choice but to study visual signatures related to daylight polarization parameters outdoors. Therefore, to study polarization signatures of targets in daylight, it is essential to have knowledge of the polarization state of the light originating from each and every point on the celestial sphere. A complete polarization profile of skylight need only include the degree of polarization and the polarization azimuth angle, since only linearly polarized light has ever been detected from skylight. Many authors give an equation for the degree of polarization due to Rayleigh scattering[10-12] in terms of the scattering angle. This paper gives an equation for both the degree of polarization and the polarization azimuth angle of skylight in terms of the position of the sun and the position of the point of observation on the celestial sphere in the horizon coordinate system. A pictorial representation of the polarization state of the sky, using the Poincare colorization methodology, is then presented. Using the colorization methodology, a visual correlation is given between the polarization states of light reflected from a target and the solar illumination producing it.

THE STOKES PARAMETERS AND THE POINCARÉ SPHERE

To determine the state of polarization corresponding to points in an image, three independent parameters must be determined for each pixel[13]. For example, the three independent parameters could be the amplitudes of the x and y components of the electric vector E_{0x} and E_{0y} and their phase difference, δ, along the optical axis. A prominent method to determine these three independent parameters, using measurable quantities, is the Stokes method[14]. This method involves measuring four intensities of a light beam. Each measurement corresponds to the intensity of the beam after it passes through each of four different filter system arrangements. The four Stokes parameters, sometimes called S_0, S_1, S_2 and S_3, are derived from these four measured intensities and form a four-element column vector in four-dimensional mathematical space. The Stokes parameters are applicable to a beam of light that is completely polarized, partially polarized or unpolarized; the beam may be monochromatic or polychromatic. The Stokes parameters for completely polarized light propagating along the +z-axis are

$$
\begin{aligned}
S_0 &= E_{0X}^2 + E_{0Y}^2 \\
S_1 &= E_{0X}^2 - E_{0Y}^2 \\
S_2 &= 2\, E_{0X}\, E_{0Y} \cos\delta \\
S_3 &= 2\, E_{0X}\, E_{0Y} \sin\delta
\end{aligned}
\qquad (1)
$$

Since the primary purpose of this work is to determine the polarization states associated with pixels in an image of a scene, we are primarily interested in the relative values of the Stokes parameters. We obtain *normalized parameters* by dividing S_0, S_1, S_2 and S_3 by S_0. A normalized Stokes vector becomes $\{1, S_1/S_0, S_2/S_0, S_3/S_0\}$. The Stokes vector of a completely polarized beam is closely related to the Poincaré sphere representation illustrated in Figure 1. Every point on the surface of the sphere corresponds to a unique state of polarization of a plane monochromatic wave and all possible states of polarization have representation on the surface of the sphere. The parameter S_0 corresponds to the radius of the sphere and the

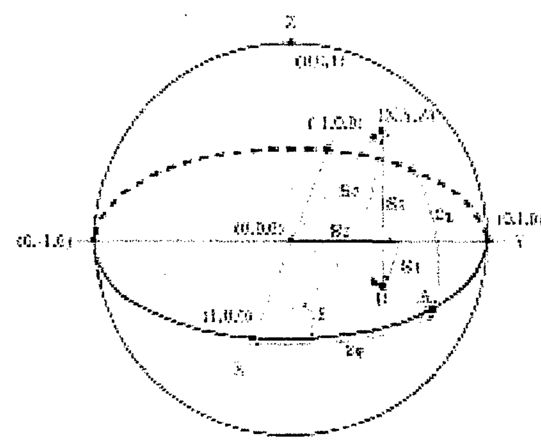

Figure 1. The relationship between the Stokes parameters and the Poncaré sphere.

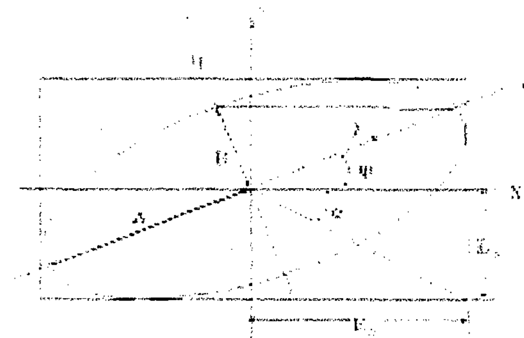

Figure 2. The polarization ellipse. The parameters ψ, χ are the polarization azimuth and ellipticity angles.

parameters S_1, S_2 and S_3 correspond to the rectangular Cartesian coordinates of a point (X,Y,Z) on the surface of the sphere. For partially polarized light the degree of polarization P is given by[15]

$$P = \frac{I_{Polarized}}{I_{Total}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \tag{2}$$

where $I_{Polarized}$ is the intensity of the polarized component and I_{Total} is the total intensity. When the Stokes parameters are normalized, the degree of polarization P becomes the radius of a sphere with center coincident to the typical Poincaré sphere of unit radius (P = 1). The inside of the Poincaré sphere can be used to represent the polarized portion of partially polarized light (P < 1). The polarization forms on the surface of the inside spheres (P < 1) are exactly the same as the polarization forms on the surface of the sphere of unit radius. The benefit of this thought resides in the ability to colorize differently the same polarization form associated with a different degree of polarization. From the geometry of Figure 1 and Eq. (2) it can be shown that for normalized Stokes parameters and for any degree of polarization

$$S_1 = P\cos 2\psi \cos 2\chi,\ S_2 = P\sin 2\psi \cos 2\chi,\ S_3 = P\sin 2\chi \tag{3}$$

In fact, Eq. (3) represents the spherical coordinates for any point on the surface or inside of the Poincaré sphere where (x, y, z) = (S_1, S_2, S_3). The polarization azimuth angle, ψ, and the ellipticity angle, χ, are also defined in Figure 2. Using Eq. (3), ψ and χ can be determined from

$$\sin 2\chi = \frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}} \qquad \text{and} \qquad \tan 2\psi = \frac{S_2}{S_1} \tag{4}$$

The parameter χ varies from +45° to -45°; it is positive for right-handed polarization forms and negative for left-handed polarization forms. The parameter ψ varies from 0° to 180°; it is 0° for horizontal polarization forms and 90° for vertical polarization forms.

ENCODING POLARIZATION PARAMETERS IN A DAYLIGHT SCENE

The Stokes parameters can easily be encoded in a daylight scene by assigning RGB values to the normalized values of S_1, S_2 and S_3 at each pixel site in the scene as follows:

$$R = \mathrm{int}[127.5\,(1 - S_1)],\ G = \mathrm{int}[127.5\,(1 - S_2)] \text{ and } B = \mathrm{int}[127.5\,(1 - S_3)] \tag{5}$$

This pseudo-color scheme closely relates to the Poincaré sphere representation of partially polarized light where each Stokes vector maps into unique RGB values for 0*P*1. Unpolarized light ($S_1 = S_2 = S_3 = 0$) corresponds to middle gray (R = G = B = 127) at the center of the sphere (see Figure 3) while the inside of the sphere relates to partially polarized light (0<P<1). In this color scheme unpolarized or weakly polarized light is middle gray or highly unsaturated in the primary colors. The surface of the sphere consists of totally polarized light with P = 1. Figure 4 gives the appearance of some polarization forms, the pseudo-colors associated with them and their position on the Poincaré sphere. Figures 5 and 6 show the top and bottom views of the colorized Poincaré sphere, which relate to right and left-handed polarization forms respectfully.

The azimuth and ellipticity polarization angles are essential parameters in obtaining a complete polarization profile. One method of displaying these calculated parameters is to assign a different color to each specific angle. The equator of the Poincaré sphere plays a very special role in our pseudo-coloring scheme. It corresponds to linearly polarized light ($S_3 = 0$) and is also used to encode the polarization azimuth and ellipticity angles in a daylight scene (see Figure 7). Using $S_0 = 1$ and $\chi = 0$ in Eq. (3) and substituting into Eq. (5) yields

$$R = \mathrm{int}[127.5\,(1 - \cos 2\psi)],\ G = \mathrm{int}[127.5\,(1 - \sin 2\psi)] \text{ and } B = 127 \tag{6}$$

Substituting χ for ψ in Eq. (6) produces a color-mapping scheme for the χ-images.

$$R = \mathrm{int}[127.5\,(1 - \cos 2\chi)],\ G = \mathrm{int}[127.5\,(1 - \sin 2\chi)] \text{ and } B = 127 \tag{7}$$

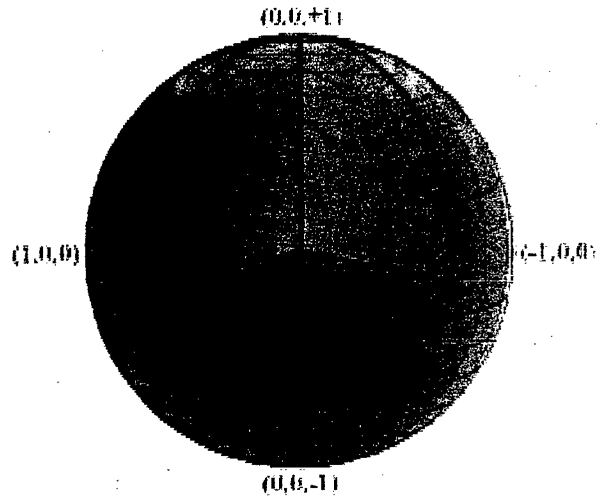

Figure 3. The colorized Poincaré sphere of unit radius. On the surface of the sphere P = 1, at the center P = 0, inside the sphere P < 1.

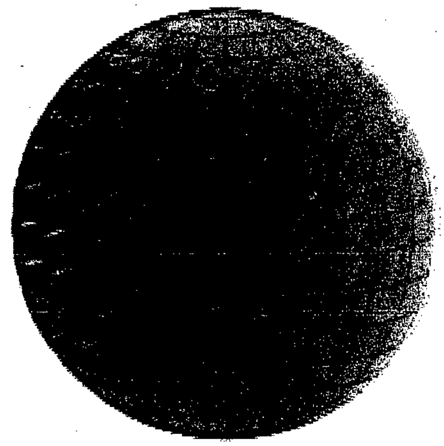

Figure 4. Pictorial representation of the polarization states corresponding to points on the surface of the colorized Poincaré sphere.

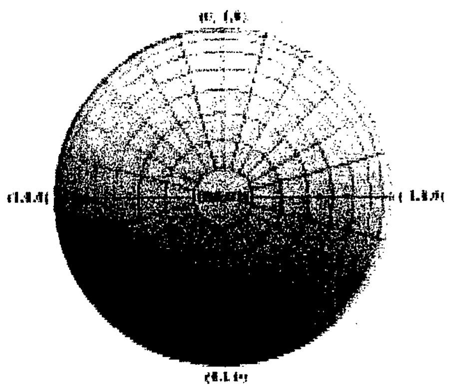

Figure 5. Top view of the colorized Poincaré sphere corresponding to right-handed polarization.

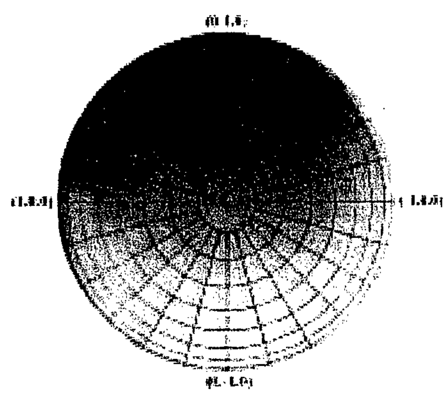

Figure 6. Bottom view of the colorized Poincaré sphere corresponding to left-handed polarization.

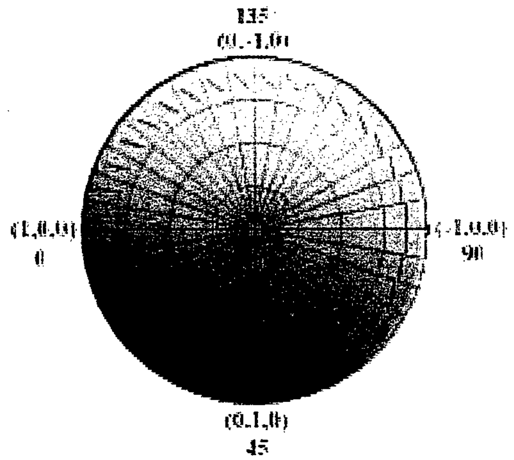

Figure 7. The Equatorial plane of the colorized Poincaré sphere.

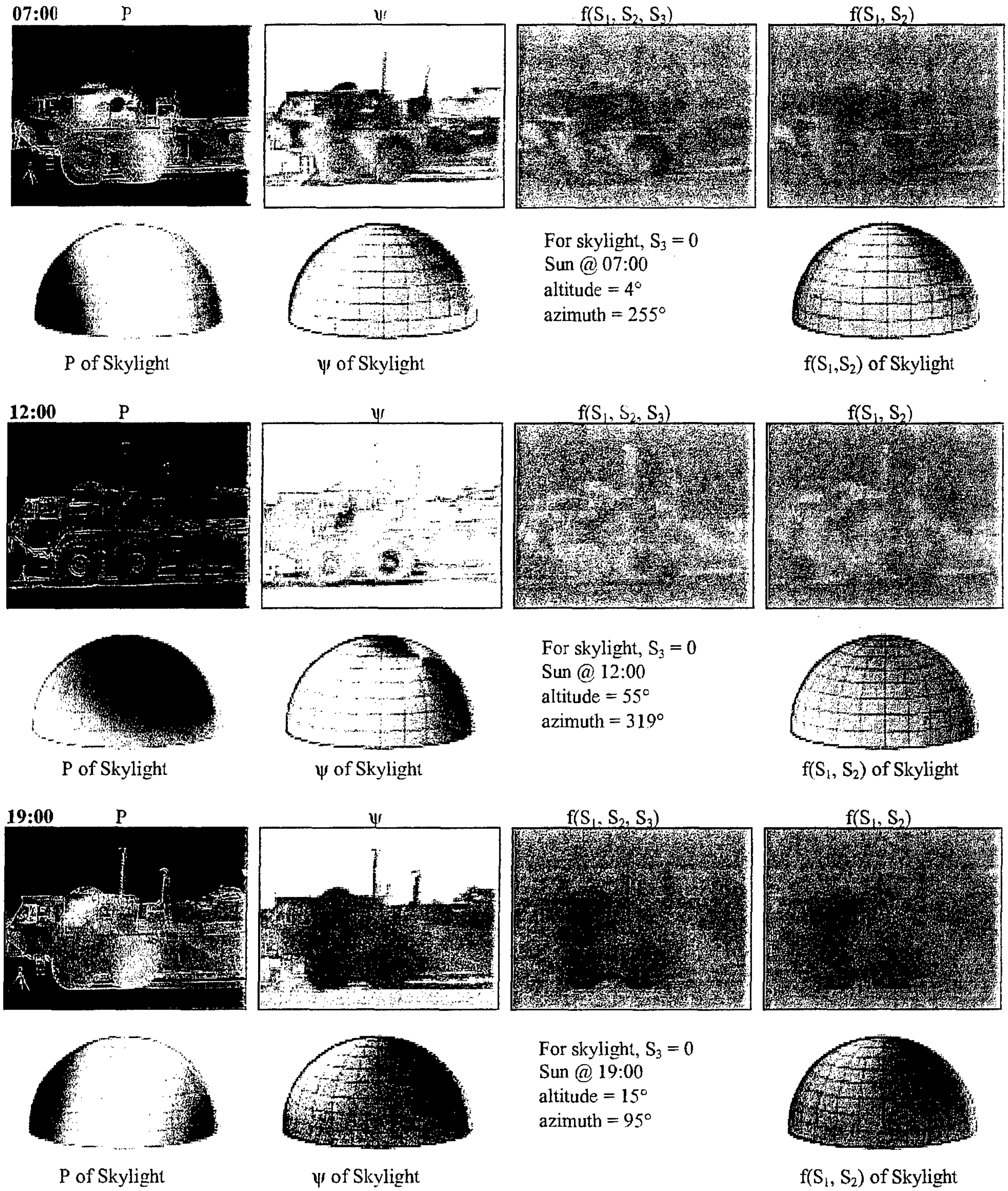

Figure 8. An example of the use of our encoding methodology to colorize the polarization parameters associated with a vehicle in daylight and polarized skylight. P = the degree of polarization, ψ = polarization azimuth angle, $f(S_1, S_2, S_3)$ and $f(S_1, S_2)$ = colorization according to Eq. (5). Azimuth convention: south = 0°, west = 90°, north = 180°, east = 270°.

There are certain cases where the Stokes parameter representation is ill conditioned. For example, as Eq. (4) shows, the azimuth angle is undefined when S_1 and S_2 are both zero (circularly polarized or unpolarized light). Also, the value of ψ and χ is undefined when $S_0 = 0$ (no light). Again, when the normalized value of S_3 approaches one (elliptically polarized light becomes nearly circular) the measured value of S_3 may exceed one due to signal to noise effects and produce an undefined χ-value ($\sin 2\chi > 1$). Since black (0,0,0), white (255,255,255) and yellow (255,255,0) are excluded from the encoding scheme of Eqs. (6-7), they are used for special conditions. Black is used when $S_0 = 0$, white for indeterminate values such as dividing by zero and for $P = 0$; yellow is used in the ψ-images whenever $\chi = \pm 45°$.

The degree of polarization, P, varies between 0 and 1. The simplest method of encoding this parameter in a daylight scene is to use the equation [pixel value] = 255 P. The black areas (pixel value = 0) in the resulting monochrome image will correspond to no polarization and the white areas (pixel value = 255) will correspond to light that is 100 percent polarized.

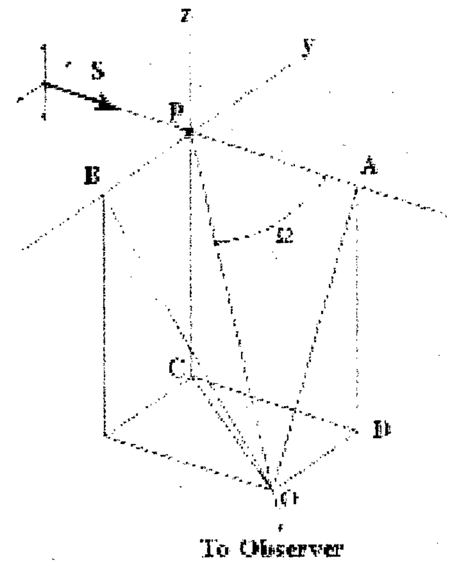

Figure 9. The scattering angle Ω.

Figure 8 gives an example of the use of our encoding methodology to colorize the polarization parameters associated with a vehicle in daylight. The images relating to Figure 8 were captured during clear skies, near sunrise, noon and sunset on April 26, 2000, at north latitude 42.5 degrees and west longitude 83 degrees. A normal to a vertical vehicle panel, which faces the digital camera, points south; the camera view is toward the north. A comprehensive diurnal study of this vehicle involved acquiring enough images to calculate the Stokes parameters every 30 minutes from sunrise to sunset. Several derived images from this study are shown in Figure 8. The notations $f(S_1, S_2, S_3)$ and $f(S_1, S_2)$ refer to the encoding scheme given in Eq. (5). The notation $f(S_1, S_2)$ implies $S_3 = 0$, which is the case for the polarization of skylight. Hence, any difference that occurs between the $f(S_1, S_2, S_3)$ and $f(S_1, S_2)$ images is due to elliptically polarized light.

The skylight images below the vehicle images in Figure 8 are pictorial representations of the polarized sky in the southern semi-hemispherical region of the celestial sphere. An Excel spreadsheet was used to create the gridlines. MatLab scripts were written to colorize the areas between the grid lines according to the encoding scheme given in Eqs. (5-6). The skylight polarization parameters were encoded into the images as they would appear looking toward the earth, as from a satellite above the earth. The values for the observed degree of polarization of skylight, along a line of sight, were obtained from[9-11]

$$P = \frac{\sin^2\Omega}{1+\cos^2\Omega} \tag{8}$$

where Ω is the scattering angle and is defined in Figure 9. Figure 9 shows a sun's ray S incident on particle P and the scattered light observed along the line of sight PO. The scattering angle, expressed in the horizon coordinate system, is given by

$$\cos\Omega = -\cos\alpha\cos\theta\cos(A-\varphi) - \sin\alpha\sin\theta \tag{9}$$

where α = the altitude of the sun and A = the azimuth of the sun, θ = the altitude of the point of observation and φ = the azimuth of the point of observation (south = 0° and lies in the + y direction, west = 90° and lies in the + x direction). According to Eqs. (8), if $\Omega = 90°$, scattered sunlight is 100 % linearly polarized at right angles to the observation plane. For angles $0° < \Omega < 90°$ and $90° < \Omega < 180°$ scattered sunlight is partially polarized. The maximum intensity of scattered sunlight occurs when $\Omega = 0°$ or $180°$, in the forward and backward direction. Figure 10 shows the geometry for finding the polarization azimuth angle produced by scattered sunlight. R is a unit position vector pointing toward the sun and Q is a unit position vector in the direction of observation. Since a tangent **T** to the polarization circle passing through the observation point is perpendicular to both **R** and **Q**, it can be calculated from the vector cross product **T** = **Q X R**. A vector **N**, perpendicular to the plane containing the z-axis and **Q**, is obtained from the vector cross product **N** = **Q X k**, where **k** is a unit

vector along the z-axis. The angle between T and N is the polarization azimuth angle ψ observed at the observation point $P_o(x_o,y_o,z_o)$ along the direction of Q and is obtained from a vector dot product. The value of ψ can be obtained from

$$\cos\psi = \frac{\sin\alpha\ \cos\theta - \sin\theta\ \cos\alpha\ \cos(\varphi - A)}{\pm\sqrt{\begin{array}{l}(\cos\theta\ \sin\alpha\ \cos\varphi\ - \sin\theta\ \cos\alpha\ \cos A)^2 + \\ (\sin\theta\ \cos\alpha\ \sin A - \cos\theta\ \sin\alpha\ \sin\varphi)^2 + \\ \cos^2\theta\ \cos^2\alpha\ \sin^2(\varphi - A)\end{array}}} \tag{10}$$

The inputs are the altitude α and the azimuth A of the sun and the altitude θ and the azimuth φ of a point of observation. Azimuths are measured from due south toward the west and around the horizon 360 degrees (south = 0°, west = 90°, north = 180°, east = 270°).

If $\psi < 90°$, a line L tangent to a skylight polarization circle and passing through the point $P_o(x_o,y_o,z_o)$, will pierce the x-y plane to the left of the vertical plane P, as shown in Figure 11; otherwise, if $\psi > 90°$, it will pierce the x-y plane to the right of plane P.

This simple picture of the polarization produced by the scattering of sunlight is incomplete for the following main reasons: (1) multiple scattering, (2) molecular anisotropy and (3) size of particles. However, Eqs. (8-10) are quite adequate in describing the polarization parameters associated with large regions of the sky.

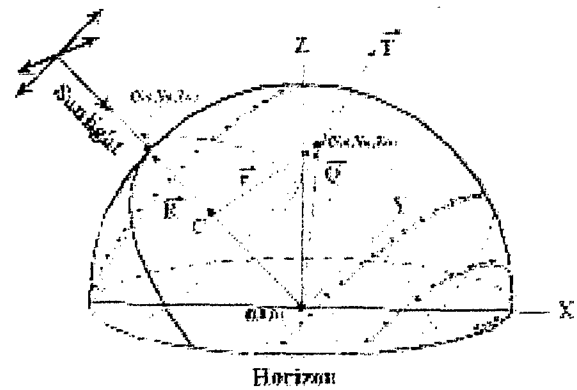

Figure 10. Geometry for finding the polarization azimuth angle produced by scattered sunlight.

ANALYSIS OF ENCODED POLARIZATION PARAMETERS IN A DAYLIGHT SCENE.

Many factors determine the nature of the reflected light from objects in a daylight scene that are recorded by a digital camera. The nature of the incident light, the nature and geometry of the material the incident light interacts with, the object-light source geometry and the spatial geometry between the camera, the object and the light source. The primary sources of light in a daylight scene are direct sunlight and skylight. Sunlight is unpolarized and skylight has polarization characteristics described by Eqs. (8-10). Commercial computer software is available to determine the position of the sun at any location and time on earth[16]. Since the view of the digital camera (the one used to acquire the images in Figure 8) was to the north and its optical axis was horizontal, the light that entered the digital camera was approximately horizontal and directed toward the south.

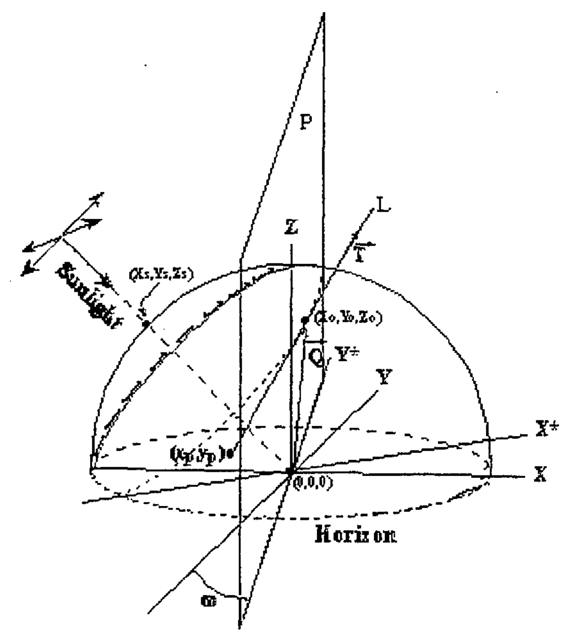

Figure 11. Geometry for finding the piercing point (x_p,y_p) in the x-y plane.

An examination of the encoding of polarization parameters in the daylight scenes of Figure 8 reveals a feature common to all images; namely, the color of the target affects the polarization state of the reflected light. The door panel of the vehicle, which lies in a vertical plane, is a good example of this phenomenon. The reflected light from those parts of the door that have the same color have approximately the same degree of polarization, azimuth angle and ellipticity angle. But, different colors on the door panel have different P, ψ and χ-values.

The P-images of Figure 8 (degree of polarization) at 07:00 and 19:00 show much less contrast and higher values of P than does the image at 12:00. At 07:00 and 19:00, the skylight images (below the vehicle images in Figure 8) show that highly polarized skylight exists at all altitudes in the southern sky, i.e., along the meridian and directly opposite the vehicle. Figure 12 shows that both edges and vertical panels can reflect highly polarized light into the camera. Since the value of P is large for both of these regions, there is little contrast between them in the P-images. The sun has yet to cross the vertical plane of the vehicle at 07:00 and is in the western sky, near the horizon, at 19:00. Neither of these positions of the sun can contribute much to producing highly polarized reflected light from the vehicle. At 12:00, the altitude and azimuth of the sun are 55° and 319° respectfully. Highly polarized skylight is near the east and western portions of the visible sky. Hence, skylight contributes very little to the degree of polarization of reflected light from vertical panels. As Figure 13 illustrates, only sunlight that reflects specularly from edges can enter the camera at this time; specular reflections from vertical panels are toward the ground. However, sunlight that reflects diffusely from both edges and vertical panels can enter the camera. In-house studies, with surfaces coated with the same material as the vehicle in Figure 8, show that the highest degree of polarization (0.55) occurs for obtuse angles between the incident light and the light reflected from the surface. Since obtuse diffuse angles of reflection for vertical panels are directed toward the ground, the edges will show a larger degree of polarization than vertical panels. This results in more contrast between the vertical panels and edges of the vehicle in the P-images.

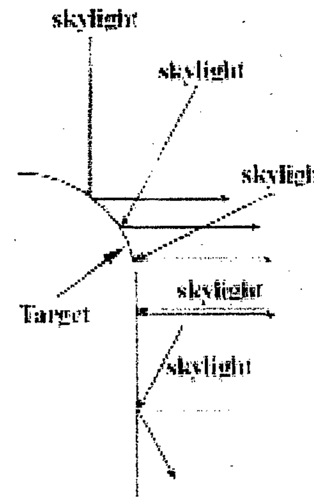

Figure 12. At 07:00 and 19:00, highly polarized skylight along the meridian is incident on the target.

In-house studies, of colored panels painted with the same material as the vehicle shown in Figure 8, show that diffuse reflections from vertical panels, for all angles of incidence, produced by *unpolarized* incident light (analogous to sunlight), have polarization azimuth angles of 90°. This implies that the electric field vectors of the reflected light are parallel to the surface. In-house studies also show that diffuse reflections, produced by *polarized* incident light (analogous to skylight), have polarization azimuth angles that are color dependent. One of the colors showed that diffuse reflections from vertical panels, produced by *polarized* incident light, for all angles of incidence, have polarization azimuth angles that are the mirror images of the azimuth angles of the incident light. In other words, with regards to reflection azimuth angles produced by polarized incident light, this particular colored target performs like a mirror. In-house studies also show that the largest degree of polarization of reflected light that originates from *unpolarized* incident light is 50% as compared to nearly 100% for *polarized* incident light. Hence, skylight will tend to dominate over sunlight, with regards to measured polarization azimuth angles, ψ, of reflected light from the vehicle. This behavior of ψ is seen in the daylight scenes of Figure 8. A comparison of the ψ-images of the vehicle and the ψ-images of skylight, for the same time of day, shows that large regions of the vehicle have reflected azimuth angles that are approximately the mirror image of the skylight azimuth angles near the meridian and near the horizon, i.e., in the direction of the digital camera. Since ψ is determined from the ratio of S_1 and S_2, as given by Eq. (4), the $f(S_1, S_2)$-images of the vehicle in Figure 8, as determined from Eq. (5), have similar colorized regions as the $f(S_1, S_2)$-images of skylight near the meridian and near the horizon, i.e., in the direction of the digital camera.

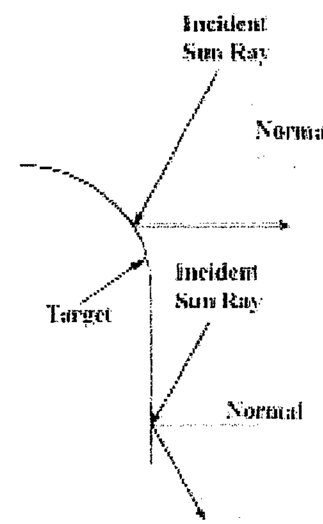

Figure 13. At 12:00, only specular reflections of sunlight from edges can enter the camera; specular reflections from vertical panels are toward the ground.

In-house studies, relating to the ellipticity angle of light reflected from panels painted with the same material as the vehicle shown in Figure 14, show the following:

1. The ellipticity angle shows some dependence on the color of the surface coating of the target.
2. For incident unpolarized light and polarized light and for diffuse reflections, the ellipticity angle χ increases with decreasing values of the degree of polarization P.

3. For diffuse reflections (scattering) and incident unpolarized light, the largest values of χ and the smallest values of P occur for backward reflections (back scattering).
4. For diffuse reflections and polarized incident light. the largest values of χ and the smallest values of P occur for incident polarization azimuth angles between 20° and 30° and for diffuse angles of incidence, I, around 100°.

In the diurnal daylight studies shown in Figure 14, the largest values of χ and the smallest values of P occur in the 12:00 image. During this time, the angle between the optical axis of the camera and the position along the horizon of the highest skylight degree of polarization was approximately 90° (east and west). Also, the skylight polarization azimuth angle during these times, near the horizon and in the east and west, was approximately 20° and 30°; in excellent agreement with the results of in-house experiments. The 07:00 images, which correspond to the sun to the east of the meridian, show primarily right-handed polarization, where as the 19:00 image, which corresponds to the sun to the west of the meridian, shows primarily left-handed polarization.

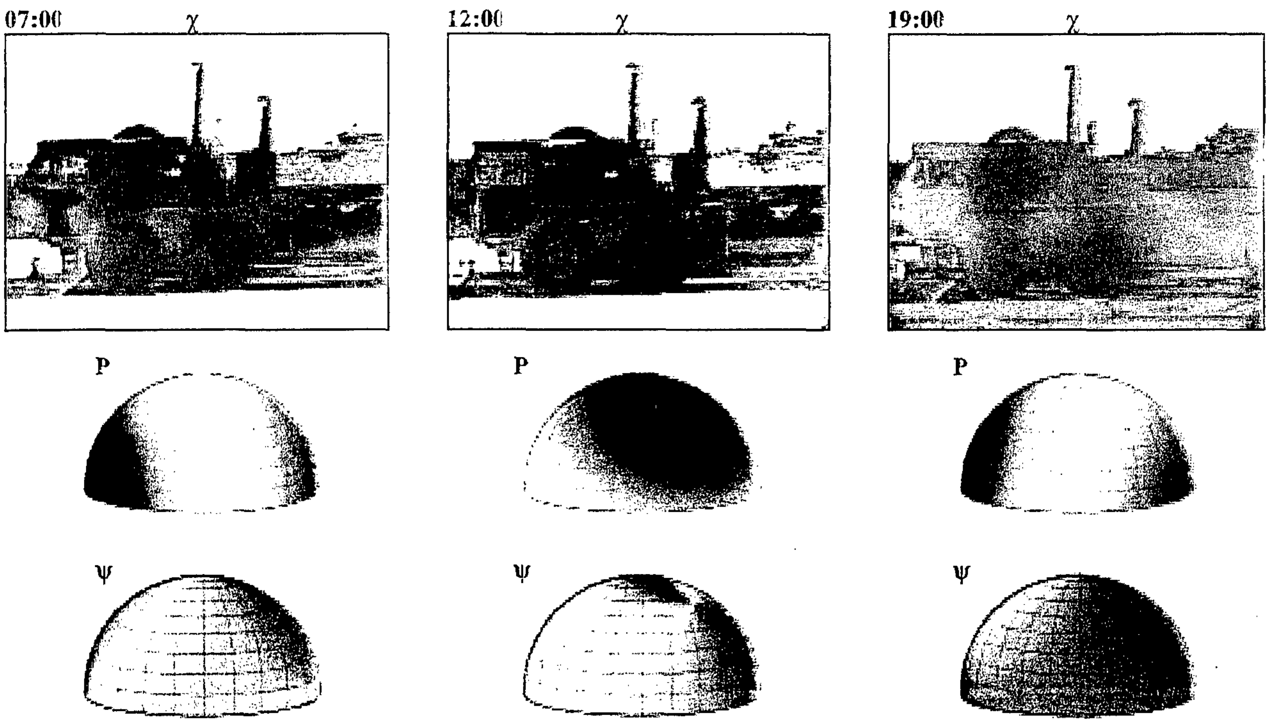

Figure 14. Encoding of the polarization ellipticity angle χ associated with a vehicle in daylight, the degree of polarization P of skylight and the polarization azimuth angle of skylight ψ.

CONCLUSION

This paper has outlined a methodology for displaying and visualizing polarization profiles of natural, daylight scenes. A pseudo color scheme, based on the Poincaré sphere, was developed for encoding various polarization parameters. Examples were given of encoding the degree of polarization and the polarization azimuth and ellipticity angles in daylight scenes at different times of the day. The scenes contained targets with surface coatings that were studied in the laboratory. In-house results, obtained from a luminance meter, were found to be in close agreement with results obtained out-doors, using a digital camera.

A mathematical model of the polarization state of skylight was given in terms of the horizon coordinate system. A pictorial representation of the polarization state of the sky was presented that used the Poincaré colorization methodology and

corresponded to the times of the sample daylight scenes. Using the Poincaré colorization methodology, a visual correlation was given between the polarization states of light reflected from a target and the solar illumination producing it.

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